

Activity 24 Piston motion

1. Points appear to form a periodic graph with sinusoidal shape.

2. By Pythagoras,

$$AP^2 = BP^2 - AB^2$$

$$= 13^2 - 5^2$$

$$AP = 12 \text{ units}$$

3. Maximum is 18 units (13+5) and minimum is 8 units.

4.

a) 10 revolutions per second

b) Period = 0.1 seconds

5. $y = 5.000 \sin(62.83x - 0.007254) + 12.49$

6.

5.000 is the amplitude of motion (maximum deviation from mean height)

62.83 is the number of cycles in 2π seconds

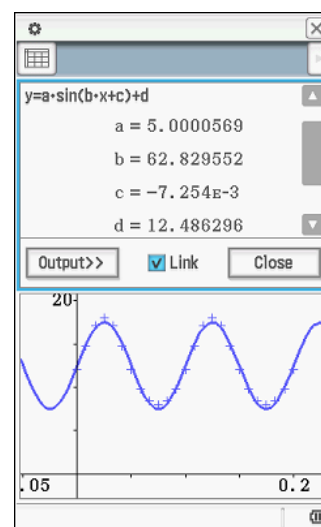
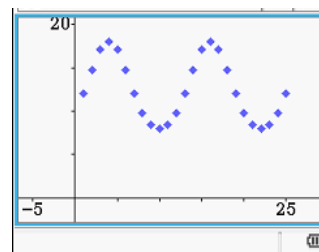
-0.007254 is the phase shift which suggests the motion starts with the piston not at mean height

12.49 is the mean height

7.

a) The crank is rotating at 10 revolutions per second which corresponds to 20π radians each second.

b) By the cosine rule, $13^2 = 5^2 + y^2 - 2 \times 5 \times y \cos\left(\frac{\pi}{2} - \theta\right)$



8. The second equation is the appropriate one.

$$y = 5 \sin(20\pi x) + \sqrt{25(\sin(20\pi x))^2 + 144}$$

The screenshot shows a CAS window with the following content:

- Equation: $13^2 = 5^2 + y^2 - 2 \cdot 5 \cdot y \cdot \cos\left(\frac{\pi}{2} - \theta\right)$
- Equation: $169 = y^2 - 10 \cdot y \cdot \sin(\theta) + 25$
- Assumption: $\text{ans} | \theta = 20\pi x$
- Equation: $169 = y^2 - 10 \cdot y \cdot \sin(20 \cdot x \cdot \pi) + 25$
- Command: $\text{Solve}(\text{ans}, y)$
- Result: $\left\{ y = 5 \cdot \sin(20 \cdot x \cdot \pi) - \sqrt{25 \cdot (\sin(20 \cdot x \cdot \pi))^2 + 144}, y = 5 \cdot \sin(20 \cdot x \cdot \pi) + \sqrt{25 \cdot (\sin(20 \cdot x \cdot \pi))^2 + 144} \right\}$

9. The graphs are similar, although the maximum and minimum values show the greatest variation. As expected the actual maximum and minimum values are 18 and 8 units. In addition, the piston actually spends less of its time towards the top of the cylinder as indicated by the “sharper” maximums and “flatter” minimums.

